Math as an Area of Knowledge Lesson Two:

Picking up on the notions of axioms and theorems, consider that:

The logical sequence of statements that lead to a theorem is called

A PROOF.

Thus, from axioms, proofs emerge.

It is important that the difference between proof and conjecture be understood:

PROOF is necessarily true;

CONJECTURE is hypothetical (and hypothesis may or may not work)

We cannot rely on conjecture; in so doing we rely on INDUCTIVE REASONING

(returning to rationalism)

INDUCTIVE is specific to general and just because there may be many specific incidents in which the hypothesis applies, it does not make the conjecture:

*Necessarily true.*

For example, consider Goldbach’s conjecture:

Every even number greater than 2 is the sum of two primes.

Let's try a few:

4=2+2

6=3+3

8=3+5

10=3+7, 5+5

12=5+7

14=3+11, 7+7

16=3+13, 5+11

18=5+13, 7+11

20=3+17, 7+13

22=3+19, 5+17, 11+11

24=5+19, 7+17, 11+13

26=3+23, 7+19, 13+13

28=5+23, 11+17

30=7+23, 11+19, 13+17

32=3+29, 13+19

34=3+31, 5+29, 11+23, 17+17

36=5+31, 7+29, 13+23, 17+19

38=7+31, 19+19

40=3+37, 11+29, 17+23

All of the even number up to 400,000,000,000 have been tested, so far, with no exceptions found.

However, just because there have been (albeit many!) inductively reasoned tests, it does not prove Goldbach’s theory as “necessarily true,” and, according to Jim Loy and his website (jimloy.com): “Incidentally, if either Goldbach Conjecture is ever proven, then that would also prove that there are infinitely many primes. Unfortunately the fact that there are infinitely many primes does not imply either Goldbach Conjecture.

Thus, we have to examine the role of CERTAINTY in mathematics:

In so doing, four other terms emerge:

ANALYTIC PROPOSITION

SYNTHETIC PROPOSITION

Analytic proposition means **true by definition**

Synthetic proposition means **not analytic, thus not true by definition**

**AND**

A PRIORI (from the earlier)

A POSTERIORI (from the later)

A priori: we know…without having had to have experienced it ourselves

A posteriori: we cannot know without benefit of experience

Quick Review: Which of these Latin terms represents empiricism?

Thus, we now need to create a chart (God help me!)

**Note: you need to take this down; we need it for Tuesday’s class**

**Personal note: chart making is hard! And so is figuring out this puppy, so take it down…. :)**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Nature of** | **Proposition** |
| **How is it** | A priori | 1 yes | 4 ? |
| **Knowable?** | A posteriori | 2 | 3 yes |

Each box explained…kind of…

**Box 1: analytic**… knowable by definition

No experience necessary!

**Box 2:** knowable by definition OR

Knowable ONLY by experience

The latter cancels the former

(OR a posteriori trumps a priori) and since something cannot be both, it is **empty**

**Box 3:** not true by definition

Cannot be known to be true independent of experience

We know certain definitions only upon the basis of experience (not necessarily our personal encounter, but the established proof…testability; documentation of someone who has indeed experienced this

In a word…**EMPIRICISM**

Box 4: a non-trivial proposition (not true by definition)

Whose truth can be known WITHOUT experience

So….not known by definition and not known by experience…hence the question mark…God bless us, everyone!